

What is Temperature?

Equilibrium Series

Instructor's Guide

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INTRO

SOPH 302

RESOURCES

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DEVELOPED BY THE TEACHING AND LEARNING LABORATORY AT MIT
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Introduction

When to Use this Video

- In SOPH 302, during the unit on Statistical Mechanics.
- Prior knowledge: second law of thermodynamics, and the definitions of microstate, macrostate, and entropy.

Learning Objectives

After watching this video students will be able to explain how the definition of the temperature arises as a derivative of entropy with respect to energy for a constant volume and constant number of moles.

Motivation

- Entropy is a notoriously difficult concept for students, while temperature seems more intuitive.
- This video uses statistical mechanics to show that entropy arises naturally when considering equilibrium conditions, and that the quantity that is constant at equilibrium for a fixed volume and fixed number of moles is the derivative of entropy with respect to energy.
- Experience allows us to connect this derivative to the quantity we expect to be the same at equilibrium: temperature.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Consider an insulated two-box system and identify how to find the energy states of box 1 and box 2 that maximizes the number of microstates with those energies.
- Differentiate the expression for the number of microstates with respect to energy that corresponds to fixed energy states of E_1 for box 1 and $E - E_1$ for box 2.
- Identify the physical quantity that best represents the derivative of entropy with respect to energy with fixed volume and number by considering properties of thermal equilibrium.

Key Information

Duration: 12:09

Narrator: Prof. Jeffrey Grossman

Materials Needed:

- Paper
- Pencil

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Video Highlights

This table outlines a collection of activities and important ideas from the video.

| Time | Feature | Comments |
|-------|--|--|
| 0:00 | Video clip comparing behavior of heated metals and heated polymers | Motivation linking the structure of materials to the behavioral relationship between entropy and temperature. |
| 1:45 | Prerequisites and Learning Objectives | What students need to know before watching the video, and what they will learn by the end of the video. |
| 2:07 | Chapter 1: Entropy Defined | |
| 3:35 | Activity | Identify the changing and unchanging macro-state variables in an insulated two-box system separated by an impermeable, conducting wall. |
| 6:12 | Energy distributions discussed | Given a fixed total energy for the sum of box 1 and box 2, the energy distributions in box 1 is explored as the number of particles increases. |
| 6:45 | Activity | Determine the energy distribution between box 1 and 2 that has the maximum number of microstates. |
| 7:11 | Activity | Carry out derivation and check work against video computation. |
| 7:35 | Entropy is defined | |
| 8:03 | Chapter 2: Entropy and Temperature | |
| 8:39 | Equilibrium defined | Discusses that at equilibrium in the 2 box system, the energies of each box are not equal. |
| 9:27 | Temperature defined | Temperature defined as the property that is equal at equilibrium, and thus is related to the derivative of entropy with respect to energy. |
| 10:56 | Boltzmann constant | Students see that Boltzmann's constant is required to convert between units of energy and units of temperature. |
| 11:11 | To Review | Video content is summarized. |

Video Summary

This video motivates the statistical mechanics approach through material structure and behavior. Entropy is introduced as a natural variable whose derivative with respect to energy is zero when the number of microstates is maximized. This derivative is identified as being the reciprocal of temperature, and Boltzmann's constant is explained.

SOPH 302 Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

The first two problems check student understanding of temperature measurement. The last problem checks student understanding of microstate and macrostates of energy configurations of noninteracting particles.



1. Suppose you have 10 people jumping on trampolines. Everyone jumps independently but at a steady jumping pace. You take a photo and measure the height of each person relative to the relaxed trampoline height.

- Must there be one person whose height off of the trampoline is equal to the average height?
- If you take another photo a short time later, will the data be the same or different? Do you expect that average height measurement to be the same or different? Why or why not?
- Explain how this height measurement is analogous to measuring temperature.




2. Suppose you have a thermometer, either digital or analog.

- What is being measured when you place the thermometer in a gas?
- What is being measured when you place the thermometer in a solid?
- Explain why you must wait a short time before you get a stable temperature measurement.
- There is a gas initially at thermal equilibrium in a large room. Your thermometer is initially very cold compared to the room. You measure the temperature at two locations on either end of the room. Do you expect the measurements to be the same or not? Explain in terms of molecules in an ideal gas.
- Why is temperature a measurement of average kinetic energy?



3. In this problem, the particles are noninteracting, and the box is thermally insulated.

- Suppose you have a box with 10 particles, each allowed an energy state of 0, 1, or 2 with a total energy of 15. Find the number of microstates corresponding to this macrostate.
- 
- Suppose you have another box with 2 particles each allowed an energy state of 0, 1, or 2 with a total energy of 2. Find the number of microstates corresponding to this macrostate.
 - Place the two boxes from (a) and (b) in thermal contact. The number of particles and allowable energy states in each box cannot change. The total energy of the two box system is 16. Find the macrostates and microstates of this system. Which macrostate corresponds to the equilibrium state? Compare the equilibrium macrostate of the composite system with those of the 2 systems in (a) and (b).

Post-Video Materials



1. Show that the entropy of the two box system is greater than or equal to the entropy of box one plus the entropy of box two. Hint: Use Stirling's formula.



2. When is the entropy of the two box system equal to the entropy of box one plus the entropy of box two?



3. True or False: Entropy is only well-defined at equilibrium. Discuss.



4. Discuss why problem two tells us that entropy is additive for 2 subsystems in thermal equilibrium. Use this to derive the second law of thermodynamics.



5. What are the dimensions of temperature? Energy? Entropy? Discuss.

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Additional Resources

Going Further

This video begins to explore the relationship between energy, entropy, and temperature in the absence of external work. The next step is to introduce ideas of work and heat change due to engineering processes, and to connect these notions to the internal energy.

This video suggests that statistical mechanics is a powerful method for uncovering the behavior of different materials using statistical information about the microscopic structure. We recommend returning to this video in EPDM 103: Structures and Materials after discussing metals, perhaps as an introduction to the differing structure of polymers.

References

A strong understanding of entropy is important to delve into the content of this video. We recommend that students review entropy by watching the Entropy video here:

- Shah, D. (Director), Rankin, J. (Producer), (2011). *Entropy*, [Video]. Available from <http://tll.mit.edu/help/entropy> (Accessed Sept 25, 2013).

The following texts are useful resources for thermodynamics, thermostatics, statistical mechanics, and a consideration of thermodynamics from a material science perspective.

- Callen, H. B. (1985). *Thermodynamics and an Introduction to Thermostatistics*. New York, NY: John Wiley & Sons, Inc.
- DeHoff, R. T. (1993). *Thermodynamics in Material Science*. New York, NY: McGraw Hill, Inc.
- McQuarrie, D. A. (1976). *Statistical Mechanics*. New York, NY: Harper & Row.